Nonparametric Causal Survival Analysis under Clustered Interference

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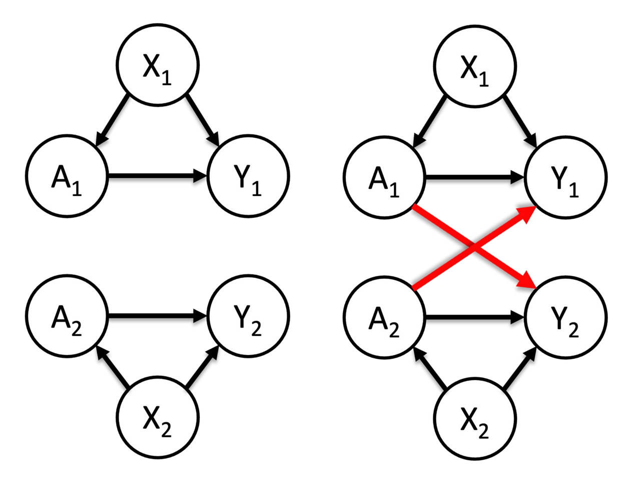
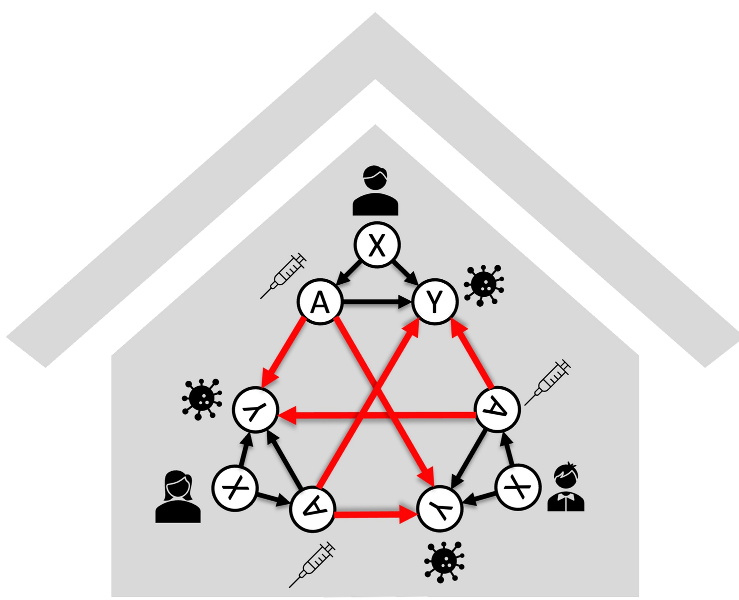
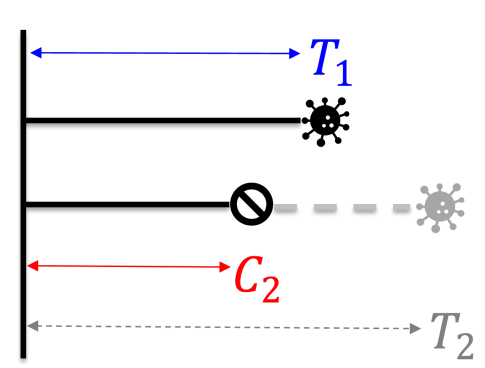
1 MOTIVATION

Clustered Interference [1]

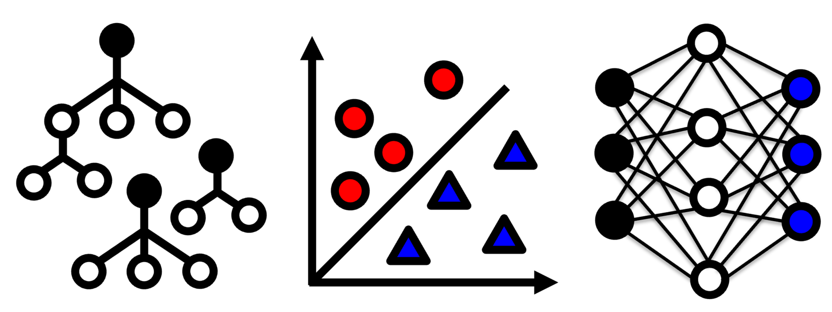




Right Censoring [2]



Nonparametric (Data-adaptive) [3]



2 BACKGROUND

Observed data

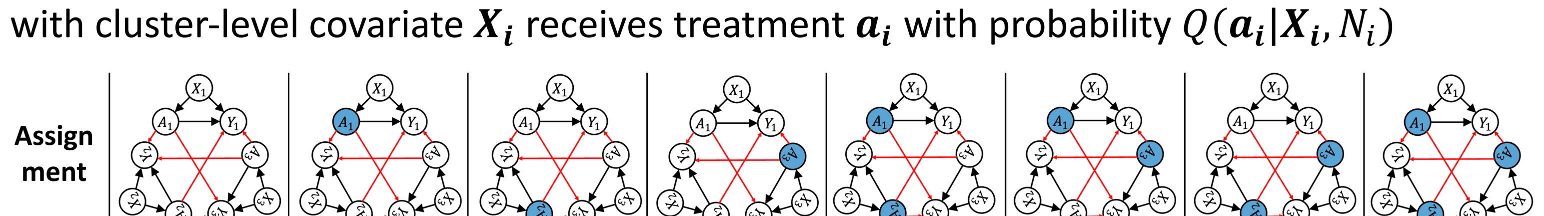
- Cluster $i \in \{1,\dots,m\}$, Unit $j \in \{1,\dots,N_i\}$
- $T_{ij} \in \mathbb{R}^+$: event time, $A_{ij} \in \{0,1\}$: treatment, $X_{ij} \in \mathbb{R}^p$: confounders
- $C_{ij} \in \mathbb{R}^+$: censoring time, $Y_{ij} = \min(T_{ij}, C_{ij})$, $\Delta_{ij} = 1(T_{ij} \le C_{ij})$

Potential outcome

- $T_{ij}(\boldsymbol{a}_i)$: Potential event time for unit j in cluster i when the cluster i receives $\boldsymbol{a}_i$
- $T_{ij}(\boldsymbol{a}_i) = T_{ij}(a_{ij}, \boldsymbol{a}_{i(-j)})$, $\boldsymbol{a}_{i(-j)} = (a_{i1},\dots,a_{i(j-1)}, a_{i(j+1)},\dots a_{iN_i})$
- No interference: $T_{ij}(a_{ij}, \boldsymbol{a}_{i(-j)}) = T_{ij}(a_{ij}, \boldsymbol{a'}_{i(-j)})$

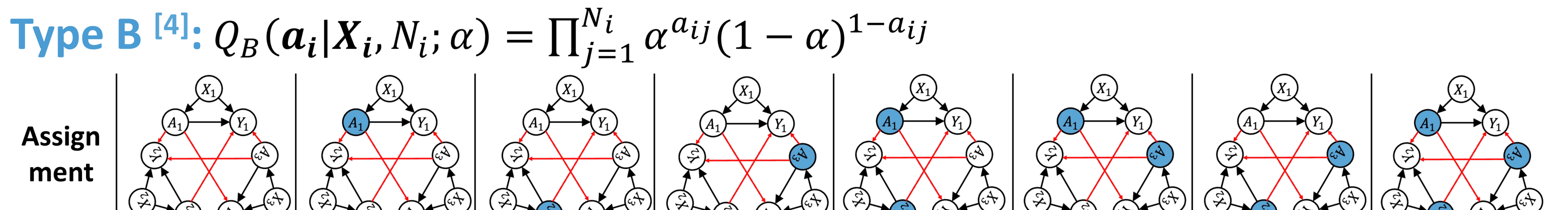
3 STOCHASTIC POLICY

Definition $Q(\cdot|\boldsymbol{X}_i, N_i)$: probability distribution on $\{0,1\}^{N_i}$ such that cluster of size N_i with cluster-level covariate $\boldsymbol{X}_i$ receives treatment $\boldsymbol{a}_i$ with probability $Q(\boldsymbol{a}_i|\boldsymbol{X}_i, N_i)$



Assignment	(graph)	(graph)	(graph)	(graph)	(graph)	(graph)	(graph)	(graph)
Prob	$Q(0,0,0\|X,N)$	$Q(1,0,0\|X,N)$	$Q(0,1,0\|X,N)$	$Q(0,0,1\|X,N)$	$Q(1,1,0\|X,N)$	$Q(1,0,1\|X,N)$	$Q(0,1,1\|X,N)$	$Q(1,1,1\|X,N)$

Type B [4]: $Q_B(\boldsymbol{a}_i|\boldsymbol{X}_i, N_i;\alpha) = \prod_{j=1}^{N_i} \alpha^{a_{ij}}(1-\alpha)^{1-a_{ij}}$



Assignment	(graph)	(graph)	(graph)	(graph)	(graph)	(graph)	(graph)	(graph)
Prob	$(1\text{-}0.7)^3$	$0.7(1\text{-}0.7)^2$	$0.7(1\text{-}0.7)^2$	$0.7(1\text{-}0.7)^2$	$0.7^2(1\text{-}0.7)$	$0.7^2(1\text{-}0.7)$	$0.7^2(1\text{-}0.7)$	0.7^3

CIPS [5]: $Q_{CIPS}(\boldsymbol{a}_i|\boldsymbol{X}_i, N_i;\delta) = \prod_{j=1}^{N_i} \pi_{ij,\delta}^{a_{ij}}(1-\pi_{ij,\delta})^{1-a_{ij}}$

- Propensity score of unit j in cluster i: $\pi_{ij} = P(A_{ij} = 1|\boldsymbol{X}_i, N_i)$
- Shifted (counterfactual) propensity score: $\pi_{ij,\delta}$ from $\frac{\pi_{ij,\delta}}{1-\pi_{ij,\delta}} = \delta \times \frac{\pi_{ij}}{1-\pi_{ij}}$

4 ESTIMANDS

- **Expected Overall Risk by time τ under policy Q**

$$\mu(\tau;Q) = \mathbb{E}\left\{\frac{1}{N_i}\sum_{j=1}^{N_i}\sum_{\mathbf{a}_i\in\mathcal{A}(N_i)} \mathbb{1}(T_{ij}(\mathbf{a}_i)\le\tau)Q(\mathbf{a}_i|\mathbf{X}_i,N_i)\right\}$$

- **Expected Risk by time τ when treated under policy Q**

$$\mu_1(\tau;Q) = \mathbb{E}\left\{\frac{1}{N_i}\sum_{j=1}^{N_i}\sum_{\mathbf{a}_{i(-j)}\in\mathcal{A}(N_i-1)} \mathbb{1}(T_{ij}(1,\mathbf{a}_{i(-j)})\le\tau)Q(\mathbf{a}_{i(-j)}|\mathbf{X}_i,N_i)\right\}$$

- $OE(\tau;Q,Q') = \mu(\tau;Q) - \mu(\tau;Q')$: compares two policies overall
  E.g. Difference in overall COVID19 risks when 50% vs. 30% of neighbors vaccinated
- $DE(\tau;Q) = \mu_1(\tau;Q) - \mu_0(\tau;Q)$: effect of treatment under policy Q
  E.g. Vaccine effect on COVID19 when 50% of neighbors vaccinated
- $SE_0(\tau;Q,Q') = \mu_0(\tau;Q) - \mu_0(\tau;Q')$: compares risk when untreated
  E.g. Unvaccinated unit's COVID19 risks when 50% vs. 30% of neighbors vaccinated

5 METHOD

1. Full data estimation equation from nonparametric EIF

$$\varphi^{F,*}_{ij}(\tau;\mathbf{Z}_i) := \sum_{\mathbf{a}_i\in\mathcal{A}(N_i)}\{w_j(\mathbf{a}_i,\mathbf{X}_i,N_i)+\phi_j(\mathbf{A}_i,\mathbf{X}_i,N_i;\mathbf{a}_i)\}\mathbb{P}(T_{ij}\le\tau|\mathbf{A}_i=\mathbf{a}_i,\mathbf{X}_i,N_i)$$
$$+\mathbb{P}(\mathbf{A}_i|\mathbf{X}_i,N_i)^{-1}w_j(\mathbf{A}_i,\mathbf{X}_i,N_i)\{\mathbb{1}(T_{ij}\le\tau)-\mathbb{P}(T_{ij}\le\tau|\mathbf{A}_i,\mathbf{X}_i,N_i)\}-\Psi(\tau;\mathbf{w})$$

2. Augmented-IPCW estimating equation

$$0 = \frac{1}{m}\sum_{i=1}^{m}\frac{1}{N_i}\sum_{j=1}^{N_i}\left[\frac{\Delta_{ij}}{S^C_{ij}(Y_{ij}|\mathbf{A}_i,\mathbf{X}_i,N_i)}\varphi^{F,*}_{ij}(\tau;\mathbf{Z}_i)+\int_0^\infty\frac{\mathbb{E}\{\varphi^{F,*}_{ij}(\tau;\mathbf{Z}_i)|T_{ij}\ge r,\mathbf{A}_i,\mathbf{X}_i,N_i\}}{S^C_{ij}(r|\mathbf{A}_i,\mathbf{X}_i,N_i)}dM^C_{ij}(r)\right]$$

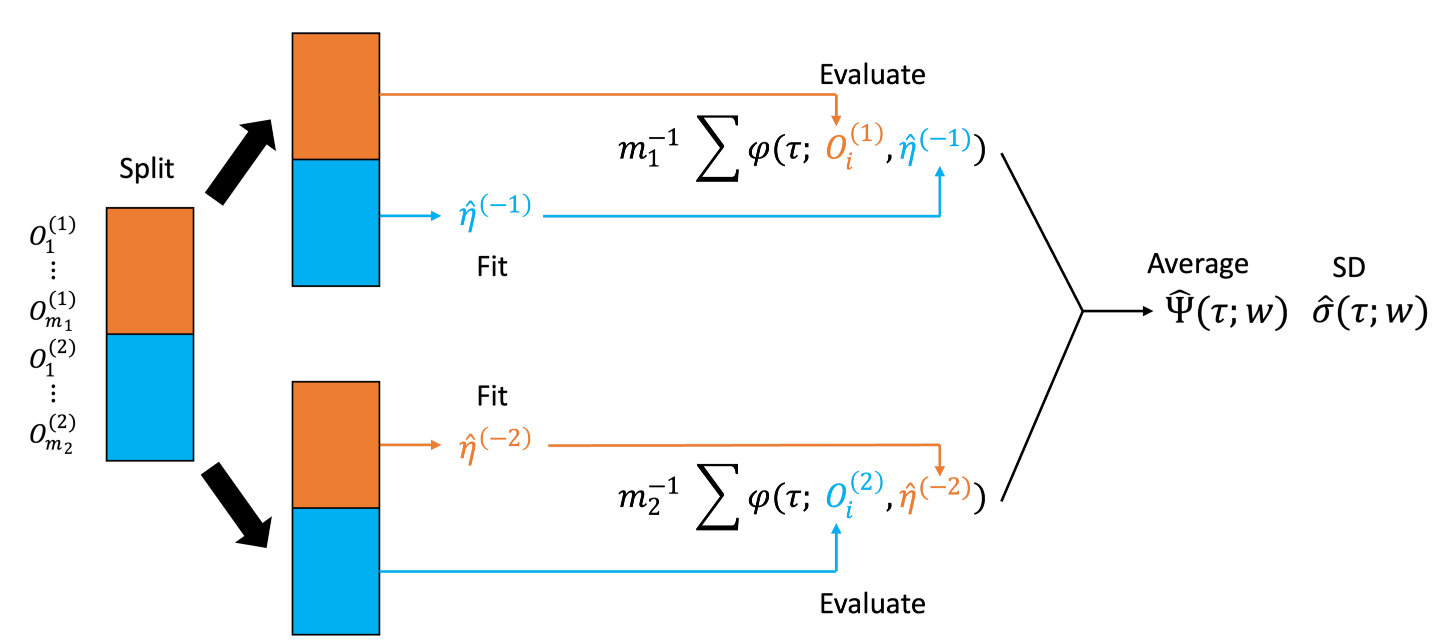
$$\varphi_{ij}(\tau;\mathbf{O}_i) = \sum_{\mathbf{a}_i\in\mathcal{A}(N_i)}\mathrm{OR}_{ij}(\tau;\mathbf{O}_i,\mathbf{a}_i) + \mathrm{IPCW\text{-}BC}ij(\tau;\mathbf{O}_i) + \mathrm{AUG}_{ij}(\tau;\mathbf{O}_i)$$

$$\mathrm{OR}_{ij}(\tau;\mathbf{O}_i,\mathbf{a}_i) = \{w_j(\mathbf{a}_i,\mathbf{X}_i,N_i)+\phi_j(\mathbf{A}_i,\mathbf{X}_i,N_i;\mathbf{a}_i)\}F^T_{ij}(\tau|\mathbf{A}_i=\mathbf{a}_i,\mathbf{X}_i,N_i),$$

$$\mathrm{IPCW\text{-}BC}_{ij}(\tau;\mathbf{O}_i) = \frac{w_j(\mathbf{A}_i,\mathbf{X}_i,N_i)}{\mathbb{P}(\mathbf{A}_i|\mathbf{X}_i,N_i)}\left\{\frac{\Delta_{ij}}{S^C_{ij}(Y_{ij}|\mathbf{A}_i,\mathbf{X}_i,N_i)}\mathbb{1}(Y_{ij}\le\tau)-F^T_{ij}(\tau|\mathbf{A}_i,\mathbf{X}_i,N_i)\right\},$$

$$\mathrm{AUG}_{ij}(\tau;\mathbf{O}_i) = \frac{w_j(\mathbf{A}_i,\mathbf{X}_i,N_i)}{\mathbb{P}(\mathbf{A}_i|\mathbf{X}_i,N_i)}\int_0^\tau\frac{S^T_{ij}(r|\mathbf{A}_i,\mathbf{X}_i,N_i)-S^T_{ij}(\tau|\mathbf{A}_i,\mathbf{X}_i,N_i)}{S^C_{ij}(r|\mathbf{A}_i,\mathbf{X}_i,N_i)S^T_{ij}(r|\mathbf{A}_i,\mathbf{X}_i,N_i)}dM^C_{ij}(r),$$

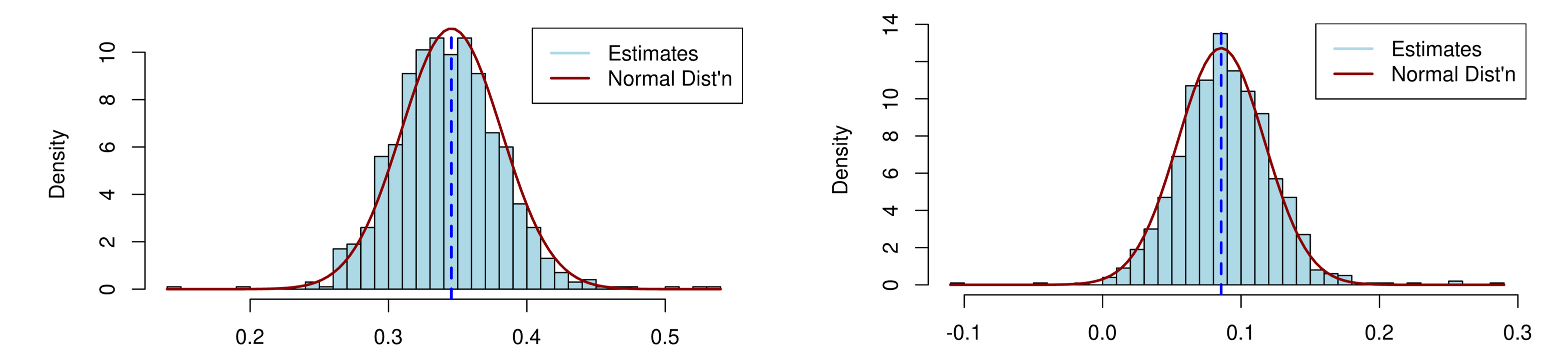
3. Sample Splitting [6] & Nonparametric Nuisance Function Estimation



6 RESULTS

Theory

- Consistent & Asymptotically Normal & Weak Convergence to Gaussian Process



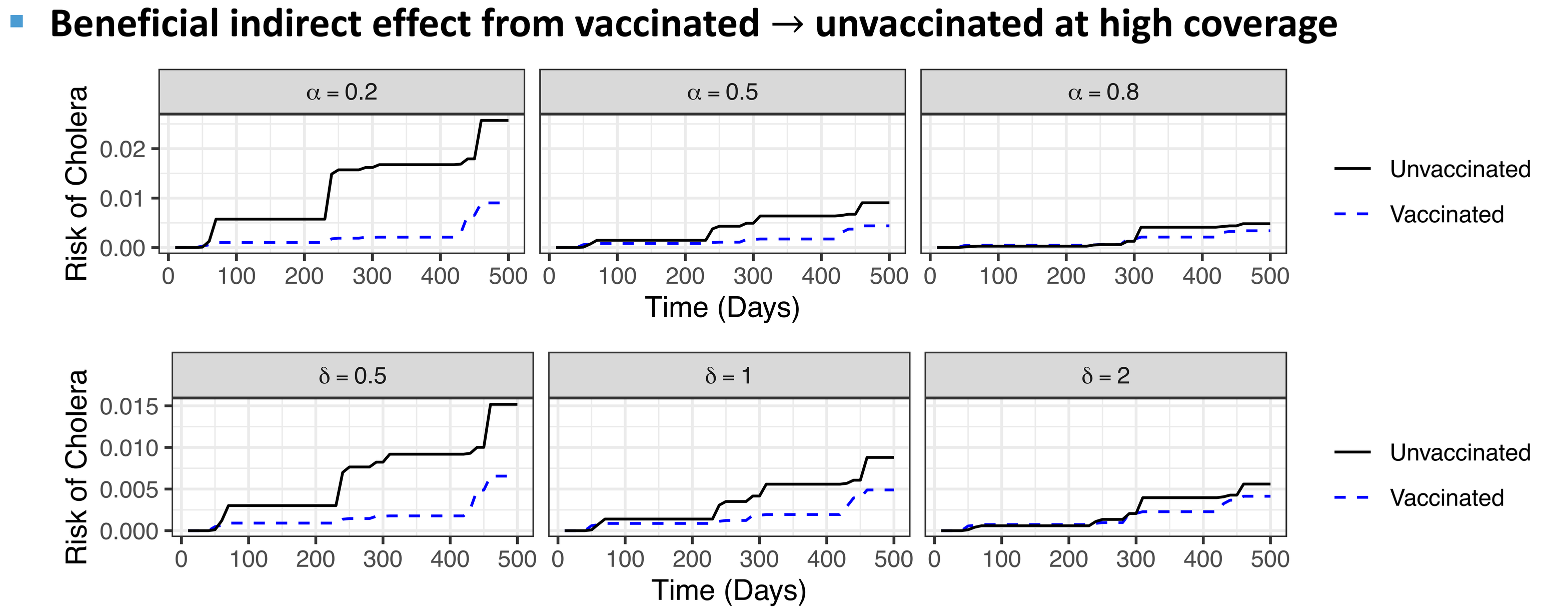
Simulation

- Flexible data-adaptive nuisance function estimation

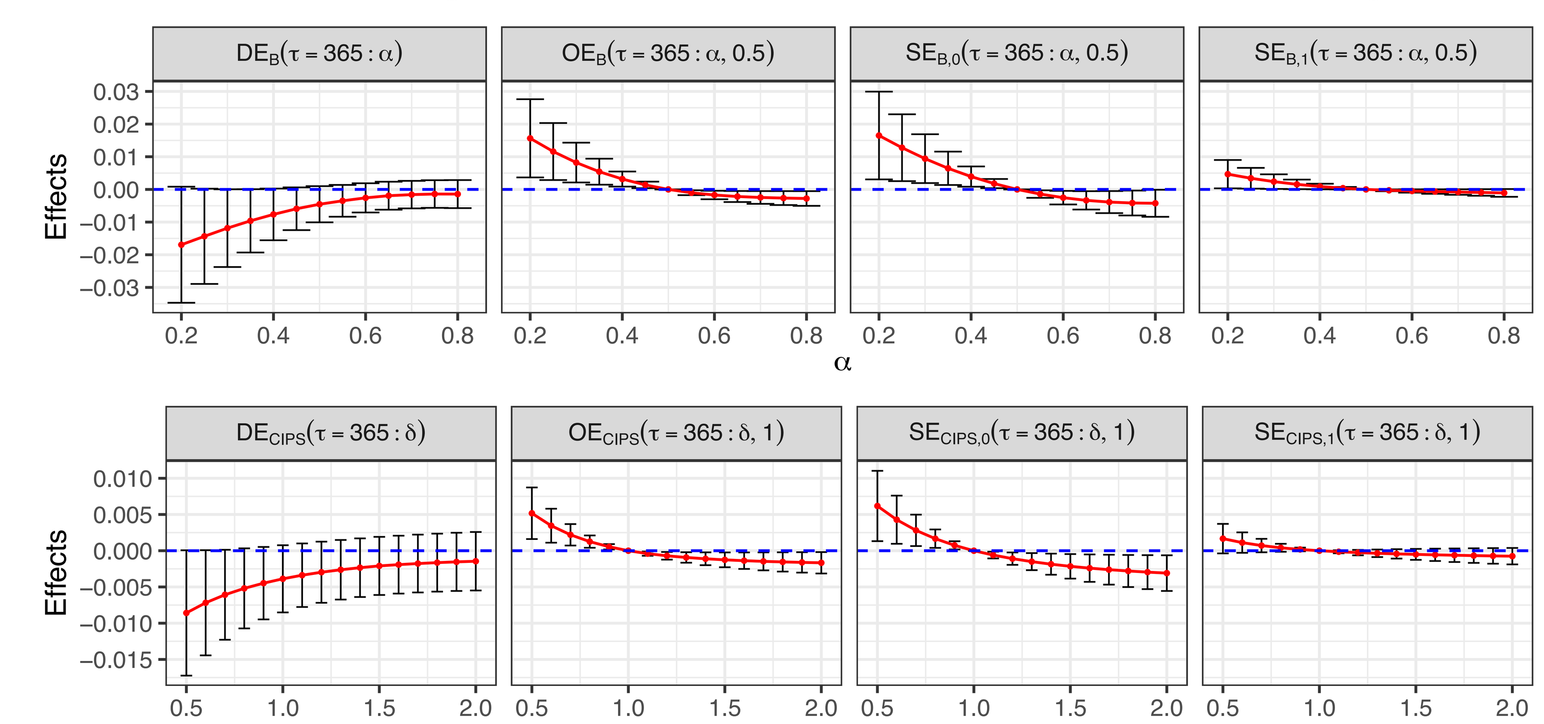
Estimand	Truth	Nonparametric (SL, RSF) Bias	RMSE	ASE	ESE	Cov %	Parametric (GLM, Cox) Bias	RMSE	ASE	ESE	Cov %	RMSE Ratio
$\mu_{\text{CIPS}}(0.3;0.5)$	0.345	-0.003	0.001	0.036	0.038	94.1%	-0.024	0.002	0.038	0.041	87.4%	0.642
$\mu_{\text{CIPS},1}(0.3;0.5)$	0.151	-0.001	0.001	0.025	0.027	93.9%	-0.016	0.001	0.027	0.028	88.3%	0.674
$\mu_{\text{CIPS},0}(0.3;0.5)$	0.524	-0.004	0.004	0.058	0.063	94.3%	-0.032	0.005	0.063	0.066	90.4%	0.744
$DE_{\text{CIPS}}(0.3;0.5)$	-0.373	0.002	0.004	0.058	0.064	94.1%	0.015	0.005	0.063	0.067	93.9%	0.853
$SE_{\text{CIPS},1}(0.3;0.5,1)$	0.021	0.001	0.001	0.021	0.024	94.9%	-0.003	0.001	0.023	0.025	95.1%	0.943
$SE_{\text{CIPS},0}(0.3;0.5,1)$	0.030	0.006	0.003	0.051	0.059	95.5%	-0.004	0.004	0.054	0.059	95.5%	0.986
$OE_{\text{CIPS}}(0.3;0.5,1)$	0.086	0.003	0.001	0.031	0.034	94.8%	-0.005	0.001	0.033	0.038	94.5%	0.800

Application (Cholera Vaccine Study)

- **Beneficial direct effect of vaccination at lower vaccine coverage**
- **Beneficial indirect effect from vaccinated → unvaccinated at high coverage**



- **Unvaccinated individuals can benefit from spillover effects from vaccinated individuals, and the magnitude of such benefit increases over vaccine coverage**



7 Discussion

- **Inference about treatment effects under clustered interference and censoring**
- **Can be applied to any stochastic treatment allocation policy**
- **Data-adaptive estimation with robust correction to yield CAN estimator**

Literature cited

1. Hudgens, M. G. and Halloran, M. E. (2008). Toward causal inference with interference. *Journal of the American Statistical Association* 103, 832–842.
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Further information

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