Nonparametric Causal Survival Analysis under Clustered Interference

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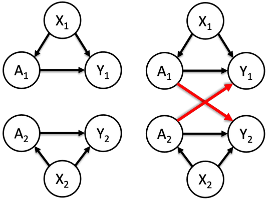
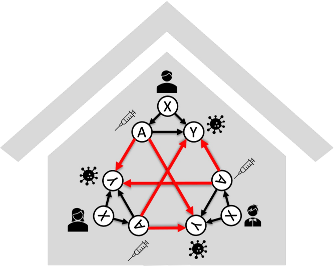
Motivation

Clustered Interference

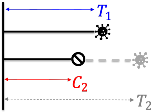


No interference **Interference**

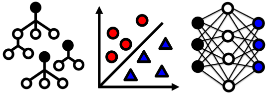


Clustered Interference

Right Censoring

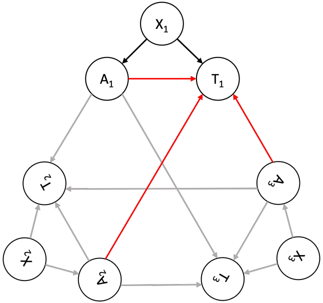


Nonparametric (Data-adaptive)



RF **SVM** **NN**

Notation



Observed data

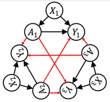
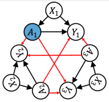
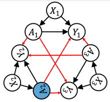
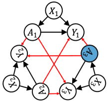
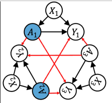
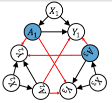
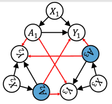
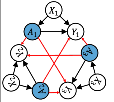
- Cluster $i \in \{1, \dots, m\}$, Unit $j \in \{1, \dots, N_i\}$
- $T_{ij} \in \mathbb{R}^+$: event time, $A_{ij} \in \{0, 1\}$: treatment, $\mathbf{X}_{ij} \in \mathbb{R}^p$: confounders, $\mathbf{Z}_i = (\mathbf{T}_i, \mathbf{A}_i, \mathbf{X}_i)$: **Full** data
- $C_{ij} \in \mathbb{R}^+$: censoring time, $Y_{ij} = \min\{T_{ij}, C_{ij}\}$: observed time, $\Delta_{ij} = \mathbb{1}(T_{ij} \leq C_{ij})$, $\mathbf{O}_i = (\mathbf{Y}_i, \mathbf{\Delta}_i, \mathbf{A}_i, \mathbf{X}_i)$: **Observed** data
- $\mathcal{A}(N_i) = \{0, 1\}^{N_i}$: set of all length N_i binary vectors

Potential outcome

- $T_{ij}(\mathbf{a}_i)$: Potential event time for unit j in cluster i when the cluster i receives treatment assignment according to $\mathbf{a}_i$
- $T_{ij}(\mathbf{a}_i) = T_{ij}(a_{ij}, \mathbf{a}_{i(-j)})$,
  $\mathbf{a}_{i(-j)} = (a_{i1}, \dots, a_{i(j-1)}, a_{i(j+1)}, \dots, a_{iN_i})$
- **No interference**: $T_{ij}(a_{ij}, \mathbf{a}_{i(-j)}) = T_{ij}(a_{ij}, \mathbf{a}'_{i(-j)})$

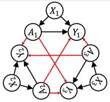
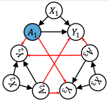
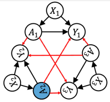
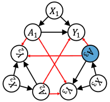
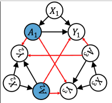
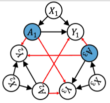
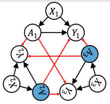
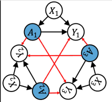
Treatment allocation policy

- Counterfactual scenario that a cluster of size N_i with cluster-level covariate $\mathbf{X}_i$ receives treatment $\mathbf{a}_i \in \mathcal{A}(N_i)$ with probability $Q(\mathbf{a}_i|\mathbf{X}_i, N_i)$
- $Q(\cdot|\mathbf{X}_i, N_i)$: probability dist'n on $\mathcal{A}(N_i)$

Assignment								
Factual P	$P(0,0,0\|\boldsymbol{X}, N)$	$P(1,0,0\|\boldsymbol{X}, N)$	$P(0,1,0\|\boldsymbol{X}, N)$	$P(0,0,1\|\boldsymbol{X}, N)$	$P(1,1,0\|\boldsymbol{X}, N)$	$P(1,0,1\|\boldsymbol{X}, N)$	$P(0,1,1\|\boldsymbol{X}, N)$	$P(1,1,1\|\boldsymbol{X}, N)$
	0.1	0.15	0.2	0.05	0.15	0.15	0.1	0.1

Treatment allocation policy

- Counterfactual scenario that a cluster of size N_i with cluster-level covariate $\mathbf{X}_i$ receives treatment $\mathbf{a}_i \in \mathcal{A}(N_i)$ with probability $Q(\mathbf{a}_i|\mathbf{X}_i, N_i)$
- $Q(\cdot|\mathbf{X}_i, N_i)$: probability dist'n on $\mathcal{A}(N_i)$

Assignment								
Factual P	$P(0,0,0\|\boldsymbol{X},N)$	$P(1,0,0\|\boldsymbol{X},N)$	$P(0,1,0\|\boldsymbol{X},N)$	$P(0,0,1\|\boldsymbol{X},N)$	$P(1,1,0\|\boldsymbol{X},N)$	$P(1,0,1\|\boldsymbol{X},N)$	$P(0,1,1\|\boldsymbol{X},N)$	$P(1,1,1\|\boldsymbol{X},N)$
	0.1	0.15	0.2	0.05	0.15	0.15	0.1	0.1
Allocation Q	$Q(0,0,0\|\boldsymbol{X},N)$	$Q(1,0,0\|\boldsymbol{X},N)$	$Q(0,1,0\|\boldsymbol{X},N)$	$Q(0,0,1\|\boldsymbol{X},N)$	$Q(1,1,0\|\boldsymbol{X},N)$	$Q(1,0,1\|\boldsymbol{X},N)$	$Q(0,1,1\|\boldsymbol{X},N)$	$Q(1,1,1\|\boldsymbol{X},N)$

Treatment allocation policy

- Counterfactual scenario that a cluster of size N_i with cluster-level covariate $\mathbf{X}_i$ receives treatment $\mathbf{a}_i \in \mathcal{A}(N_i)$ with probability $Q(\mathbf{a}_i | \mathbf{X}_i, N_i)$
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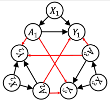
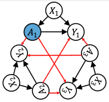
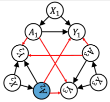
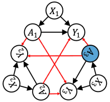
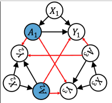
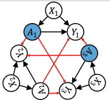
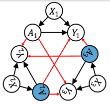
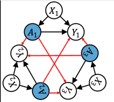
Assignment								
Factual P	$P(0,0,0 \mid \boldsymbol{X}, N)$	$P(1,0,0 \mid \boldsymbol{X}, N)$	$P(0,1,0 \mid \boldsymbol{X}, N)$	$P(0,0,1 \mid \boldsymbol{X}, N)$	$P(1,1,0 \mid \boldsymbol{X}, N)$	$P(1,0,1 \mid \boldsymbol{X}, N)$	$P(0,1,1 \mid \boldsymbol{X}, N)$	$P(1,1,1 \mid \boldsymbol{X}, N)$
	0.1	0.15	0.2	0.05	0.15	0.15	0.1	0.1
Allocation Q	$Q(0,0,0 \mid \boldsymbol{X}, N)$	$Q(1,0,0 \mid \boldsymbol{X}, N)$	$Q(0,1,0 \mid \boldsymbol{X}, N)$	$Q(0,0,1 \mid \boldsymbol{X}, N)$	$Q(1,1,0 \mid \boldsymbol{X}, N)$	$Q(1,0,1 \mid \boldsymbol{X}, N)$	$Q(0,1,1 \mid \boldsymbol{X}, N)$	$Q(1,1,1 \mid \boldsymbol{X}, N)$
Treat All	0	0	0	0	0	0	0	**1**

Treat All: $Q_{\text{All}}(\mathbf{a}_i | \mathbf{X}_i, N_i) = \prod_{j=1}^{N_i} \mathbb{1}(a_{ij} = 1)$

Treatment allocation policy

- Counterfactual scenario that a cluster of size N_i with cluster-level covariate $\mathbf{X}_i$ receives treatment $\mathbf{a}_i \in \mathcal{A}(N_i)$ with probability $Q(\mathbf{a}_i|\mathbf{X}_i, N_i)$
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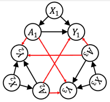
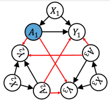
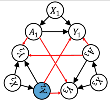
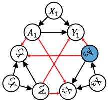
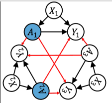
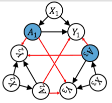
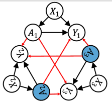
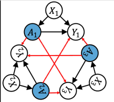
Assignment								
Factual P	$P(0,0,0|\boldsymbol{X},N)$	$P(1,0,0|\boldsymbol{X},N)$	$P(0,1,0|\boldsymbol{X},N)$	$P(0,0,1|\boldsymbol{X},N)$	$P(1,1,0|\boldsymbol{X},N)$	$P(1,0,1|\boldsymbol{X},N)$	$P(0,1,1|\boldsymbol{X},N)$	$P(1,1,1|\boldsymbol{X},N)$
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Allocation Q	$Q(0,0,0|\boldsymbol{X},N)$	$Q(1,0,0|\boldsymbol{X},N)$	$Q(0,1,0|\boldsymbol{X},N)$	$Q(0,0,1|\boldsymbol{X},N)$	$Q(1,1,0|\boldsymbol{X},N)$	$Q(1,0,1|\boldsymbol{X},N)$	$Q(0,1,1|\boldsymbol{X},N)$	$Q(1,1,1|\boldsymbol{X},N)$
Treat All	0	0	0	0	0	0	0	**1**
Treat None	**1**	0	0	0	0	0	0	0

Treat None: $Q_{\text{None}}(\mathbf{a}_i|\mathbf{X}_i, N_i) = \prod_{j=1}^{N_i} \mathbb{1}(a_{ij} = 0)$

Treatment allocation policy

- Counterfactual scenario that a cluster of size N_i with cluster-level covariate $\mathbf{X}_i$ receives treatment $\mathbf{a}_i \in \mathcal{A}(N_i)$ with probability $Q(\mathbf{a}_i | \mathbf{X}_i, N_i)$
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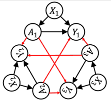
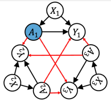
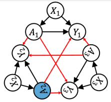
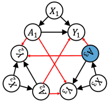
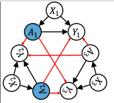
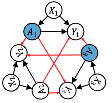
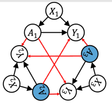
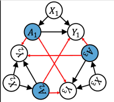
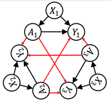
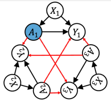
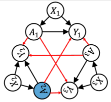
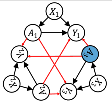
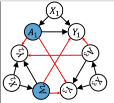
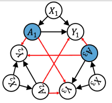
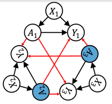
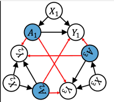
Assignment								
Factual P	$P(0,0,0 \vert \boldsymbol{X}, N)$	$P(1,0,0 \vert \boldsymbol{X}, N)$	$P(0,1,0 \vert \boldsymbol{X}, N)$	$P(0,0,1 \vert \boldsymbol{X}, N)$	$P(1,1,0 \vert \boldsymbol{X}, N)$	$P(1,0,1 \vert \boldsymbol{X}, N)$	$P(0,1,1 \vert \boldsymbol{X}, N)$	$P(1,1,1 \vert \boldsymbol{X}, N)$
	0.1	0.15	0.2	0.05	0.15	0.15	0.1	0.1
Allocation Q	$Q(0,0,0 \vert \boldsymbol{X}, N)$	$Q(1,0,0 \vert \boldsymbol{X}, N)$	$Q(0,1,0 \vert \boldsymbol{X}, N)$	$Q(0,0,1 \vert \boldsymbol{X}, N)$	$Q(1,1,0 \vert \boldsymbol{X}, N)$	$Q(1,0,1 \vert \boldsymbol{X}, N)$	$Q(0,1,1 \vert \boldsymbol{X}, N)$	$Q(1,1,1 \vert \boldsymbol{X}, N)$
Treat All	0	0	0	0	0	0	0	**1**
Treat None	**1**	0	0	0	0	0	0	0
Type B	$(1\text{-}0.7)^3$	$0.7(1\text{-}0.7)^2$	$0.7(1\text{-}0.7)^2$	$0.7(1\text{-}0.7)^2$	$0.7^2\,(1\text{-}0.7)$	$0.7^2\,(1\text{-}0.7)$	$0.7^2\,(1\text{-}0.7)$	0.7^3

Type B: $Q_{\text{B}}(\mathbf{a}_i | \mathbf{X}_i, N_i; \alpha) = \prod_{j=1}^{N_i} \alpha^{a_{ij}} (1-\alpha)^{1-a_{ij}}$

Treatment allocation policy

- Counterfactual scenario that a cluster of size N_i with cluster-level covariate $\mathbf{X}_i$ receives treatment $\mathbf{a}_i \in \mathcal{A}(N_i)$ with probability $Q(\mathbf{a}_i|\mathbf{X}_i, N_i)$
- $Q(\cdot|\mathbf{X}_i, N_i)$: probability dist'n on $\mathcal{A}(N_i)$

Assignment								
Factual P	$P(0,0,0\|\boldsymbol{X},N)$	$P(1,0,0\|\boldsymbol{X},N)$	$P(0,1,0\|\boldsymbol{X},N)$	$P(0,0,1\|\boldsymbol{X},N)$	$P(1,1,0\|\boldsymbol{X},N)$	$P(1,0,1\|\boldsymbol{X},N)$	$P(0,1,1\|\boldsymbol{X},N)$	$P(1,1,1\|\boldsymbol{X},N)$
	0.1	0.15	0.2	0.05	0.15	0.15	0.1	0.1
Allocation Q	$Q(0,0,0\|\boldsymbol{X},N)$	$Q(1,0,0\|\boldsymbol{X},N)$	$Q(0,1,0\|\boldsymbol{X},N)$	$Q(0,0,1\|\boldsymbol{X},N)$	$Q(1,1,0\|\boldsymbol{X},N)$	$Q(1,0,1\|\boldsymbol{X},N)$	$Q(0,1,1\|\boldsymbol{X},N)$	$Q(1,1,1\|\boldsymbol{X},N)$
Treat All	0	0	0	0	0	0	0	**1**
Treat None	**1**	0	0	0	0	0	0	0
Type B	$(1\text{-}0.7)^3$	$0.7(1\text{-}0.7)^2$	$0.7(1\text{-}0.7)^2$	$0.7(1\text{-}0.7)^2$	$0.7^2(1\text{-}0.7)$	$0.7^2(1\text{-}0.7)$	$0.7^2(1\text{-}0.7)$	0.7^3
TPB	**0**	**0**	**0**	**0**	0.3	0.3	0.2	0.2

Treatment Proportion Bound: $Q_{\text{TPB}}(\mathbf{a}_i|\mathbf{X}_i, N_i; \rho) = \mathbb{1}(\overline{\mathbf{a}}_i \geq \rho)\mathbb{P}(\mathbf{a}_i|\mathbf{X}_i, N_i)/\mathbb{P}(\overline{\mathbf{A}}_i \geq \rho|\mathbf{X}_i, N_i)$

Estimands

- Expected overall risk by time τ under policy Q

$$\mu(\tau; Q) = \mathbb{E}\left\{N_i^{-1}\sum_{j=1}^{N_i}\sum_{\mathbf{a}_i\in\mathcal{A}(N_i)}\mathbb{1}(T_{ij}(\mathbf{a}_i)\leq\tau)Q(\mathbf{a}_i|\mathbf{X}_i, N_i)\right\}$$

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- Expected risk by time τ **when treated** under policy Q

$$\mu_1(\tau; Q) = \mathbb{E}\Big\{N_i^{-1} \sum_{j=1}^{N_i} \sum_{\mathbf{a}_{i(-j)} \in \mathcal{A}(N_i-1)} \mathbb{1}(T_{ij}(1, \mathbf{a}_{i(-j)}) \leq \tau) Q(\mathbf{a}_{i(-j)} | \mathbf{X}_i, N_i)\Big\}$$

Estimands

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$$\mu(\tau; Q) = \mathbb{E}\left\{N_i^{-1} \sum_{j=1}^{N_i} \sum_{\mathbf{a}_i \in \mathcal{A}(N_i)} \mathbb{1}(T_{ij}(\mathbf{a}_i) \leq \tau) Q(\mathbf{a}_i | \mathbf{X}_i, N_i)\right\}$$

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  - $Q(\mathbf{a}_{i(-j)} | \mathbf{X}_i, N_i) = Q(1, \mathbf{a}_{i(-j)} | \mathbf{X}_i, N_i) + Q(0, \mathbf{a}_{i(-j)} | \mathbf{X}_i, N_i)$ probability of all units in cluster i other than j receiving treatment $\mathbf{a}_{i(-j)}$ under policy Q.

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  - No interference: $T_{ij}(1, \mathbf{a}_{i(-j)}) \equiv T_{ij}(1) \Rightarrow \mu_1(\tau; Q) \equiv \mathbb{E}\Big\{N_i^{-1} \sum_{j=1}^{N_i} Y_{ij}(1)\Big\}$

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- Expected risk by time τ **when untreated** under policy Q

$$\mu_0(\tau; Q) = \mathbb{E}\Big\{N_i^{-1} \sum_{j=1}^{N_i} \sum_{\mathbf{a}_{i(-j)} \in \mathcal{A}(N_i-1)} \mathbb{1}(T_{ij}(0, \mathbf{a}_{i(-j)}) \leq \tau) Q(\mathbf{a}_{i(-j)} | \mathbf{X}_i, N_i)\Big\}$$

Estimands: Causal effects

- $DE(\tau; Q) = \mu_1(\tau; Q) - \mu_0(\tau; Q)$: effect of treatment under policy Q
  - Vaccine effect on COVID19 risk by one year when 50% of neighbors vaccinated

Estimands: Causal effects

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- $OE(\tau; Q, Q') = \mu(\tau; Q) - \mu(\tau; Q')$: compares two policies Q and Q' overall
  - ▷ Difference in overall COVID19 risk by one year when 50% vs. 30% of neighbors vaccinated

Estimands: Causal effects

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- $OE(\tau; Q, Q') = \mu(\tau; Q) - \mu(\tau; Q')$: compares two policies Q and Q' overall
  - ▷ Difference in overall COVID19 risk by one year when 50% vs. 30% of neighbors vaccinated
- $SE_0(\tau; Q, Q') = \mu_0(\tau; Q) - \mu_0(\tau; Q')$: compares risks when untreated
  - ▷ **Unvaccinated** individual's COVID19 risks by one year when 50% vs. 30% of neighbors vaccinated

Assumptions and Identifiability

(A1) *Consistency*: $T_{ij} = \sum_{\mathbf{a}_i \in \mathcal{A}(N_i)} T_{ij}(\mathbf{a}_i)\mathbb{1}(\mathbf{A}_i = \mathbf{a}_i)$

(A2) *Conditional Exchangeability*: $T_{ij}(\mathbf{a}_i) \perp\!\!\!\perp \mathbf{A}_i | \mathbf{X}_i, N_i$ for all $\mathbf{a}_i \in \mathcal{A}(N_i)$

(A3) *Positivity*: $\mathbb{P}(A_{ij} = 1 | \mathbf{X}_i, N_i) \in (c, 1 - c)$ for some $c \in (0, 1)$

(A4) *Conditional Independent Censoring*: $T_{ij} \perp\!\!\!\perp C_{ij} | \mathbf{A}_i, \mathbf{X}_i, N_i$

(A5) *Noncensoring Positivity*: $\mathbb{P}(\Delta_{ij} = 1 | \mathbf{A}_i, \mathbf{X}_i, N_i) > 0$

(A6) *Finite cluster size*: $\mathbb{P}(N_i \leq n_{\max}) = 1$ for some $n_{\max} \in \mathbb{N}$

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Identifiability

- Using Full data: $\Psi(\tau; \mathbf{w}) = \mathbb{E}\left\{\sum_{\mathbf{a}_i \in \mathcal{A}(N_i)} \mathbf{w}(\mathbf{a}_i, \mathbf{X}_i, N_i)^\top \left[\mathbb{P}\left(T_{ij} \leq \tau | \mathbf{A}_i = \mathbf{a}_i, \mathbf{X}_i, N_i\right)\right]_{j=1}^{N_i}\right\}$

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- Using Observed data:

$$\Psi(\tau; \mathbf{w}) = \mathbb{E}\left[\sum_{\mathbf{a}_i \in \mathcal{A}(N_i)} \mathbf{w}(\mathbf{a}_i, \mathbf{X}_i, N_i)^\top \left[\mathbb{E}\left\{\frac{\Delta_{ij} \mathbb{1}(Y_{ij} \leq \tau)}{S^C_{ij}(Y_{ij} | \mathbf{A}_i, \mathbf{X}_i, N_i)} \middle| \mathbf{A}_i = \mathbf{a}_i, \mathbf{X}_i, N_i\right\}\right]_{j=1}^{N_i}\right]$$

Full data estimating equation - Nonparametric EIF

- EIF of $\Psi(\tau; \mathbf{w})$ using full data:

$$\varphi^{F,*}(\tau; \mathbf{Z}_i) = \frac{1}{N_i} \sum_{j=1}^{N_i} \varphi_{ij}^{F,*}(\tau; \mathbf{Z}_i) = \frac{1}{N_i} \sum_{j=1}^{N_i} \left[\sum_{\mathbf{a}_i \in \mathcal{A}(N_i)} \mathrm{OR}_{ij}(\tau; \mathbf{Z}_i, \mathbf{a}_i) + \mathrm{BC}_{ij}(\tau; \mathbf{Z}_i) - \Psi(\tau; \mathbf{w}) \right]$$

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$$\mathrm{OR}_{ij}(\tau; \mathbf{Z}_i, \mathbf{a}_i) = \left\{ w_j(\mathbf{a}_i, \mathbf{X}_i, N_i) + \phi_j(\mathbf{A}_i, \mathbf{X}_i, N_i; \mathbf{a}_i) \right\} F_{ij}^T(\tau | \mathbf{A}_i = \mathbf{a}_i, \mathbf{X}_i, N_i),$$

$$\mathrm{BC}_{ij}(\tau; \mathbf{Z}_i) = \frac{w_j(\mathbf{A}_i, \mathbf{X}_i, N_i)}{\mathbb{P}(\mathbf{A}_i | \mathbf{X}_i, N_i)} \left\{ \mathbb{1}(T_{ij} \le \tau) - F_{ij}^T(\tau | \mathbf{A}_i, \mathbf{X}_i, N_i) \right\}$$

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$$\Longrightarrow \widehat{\Psi}^F(\tau; \mathbf{w}) = \frac{1}{m}\sum_{i=1}^{m} \varphi^F(\tau; \mathbf{Z}_i) = \frac{1}{m}\sum_{i=1}^{m}\frac{1}{N_i}\sum_{j=1}^{N_i} \varphi_{ij}^F(\tau; \mathbf{Z}_i)$$

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$$\mathrm{OR}_{ij}(\tau;\mathbf{Z}_i,\mathbf{a}_i) = \left\{w_j(\mathbf{a}_i,\mathbf{X}_i,N_i) + \phi_j(\mathbf{A}_i,\mathbf{X}_i,N_i;\mathbf{a}_i)\right\}F_{ij}^{T}(\tau|\mathbf{A}_i=\mathbf{a}_i,\mathbf{X}_i,N_i),$$

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- EIF of $w(\mathbf{a},\mathbf{x},n)$: $\varphi^{*}_{w(\mathbf{a},\mathbf{x},n)}(\mathbf{O}_i) = \{\mathbb{1}(\mathbf{X}_i=\mathbf{x},N_i=n)/d\mathbb{P}(\mathbf{x},n)\}\phi(\mathbf{A}_i,\mathbf{X}_i,N_i;\mathbf{a})$

Observed data estimating equation - Robust correction

- Under censoring, employ IPCW and robust correction accounting for the censoring process

$$0 = \frac{1}{m}\sum_{i=1}^{m}\frac{1}{N_i}\sum_{j=1}^{N_i}\left[\frac{\Delta_{ij}}{S_{ij}^{C}(Y_{ij}|\mathbf{A}_i,\mathbf{X}_i,N_i)}\varphi_{ij}^{F,*}(\tau;\mathbf{Z}_i)+\int_0^\infty\frac{\mathbb{E}\{\varphi_{ij}^{F,*}(\tau;\mathbf{Z}_i)|\,T_{ij}\geq r,\mathbf{A}_i,\mathbf{X}_i,N_i\}}{S_{ij}^{C}(r|\mathbf{A}_i,\mathbf{X}_i,N_i)}dM_{ij}^{C}(r)\right]$$

Observed data estimating equation - Robust correction

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$$\implies \widehat{\Psi}(\tau;\mathbf{w}) = \frac{1}{m}\sum_{i=1}^{m}\varphi(\tau;\mathbf{O}_i) = \frac{1}{m}\sum_{i=1}^{m}\frac{1}{N_i}\sum_{j=1}^{N_i}\varphi_{ij}(\tau;\mathbf{O}_i)$$

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$$\varphi_{ij}(\tau;\mathbf{O}_i) = \sum_{\mathbf{a}_i\in\mathcal{A}(N_i)}\mathrm{OR}_{ij}(\tau;\mathbf{O}_i,\mathbf{a}_i) + \text{IPCW-BC}_{ij}(\tau;\mathbf{O}_i) + \mathrm{AUG}_{ij}(\tau;\mathbf{O}_i)$$

Observed data estimating equation - Robust correction

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$$\mathrm{OR}_{ij}(\tau;\mathbf{O}_i,\mathbf{a}_i) = \left\{w_j(\mathbf{a}_i,\mathbf{X}_i,N_i) + \phi_j(\mathbf{A}_i,\mathbf{X}_i,N_i;\mathbf{a}_i)\right\}F_{ij}^T(\tau|\mathbf{A}_i=\mathbf{a}_i,\mathbf{X}_i,N_i),$$

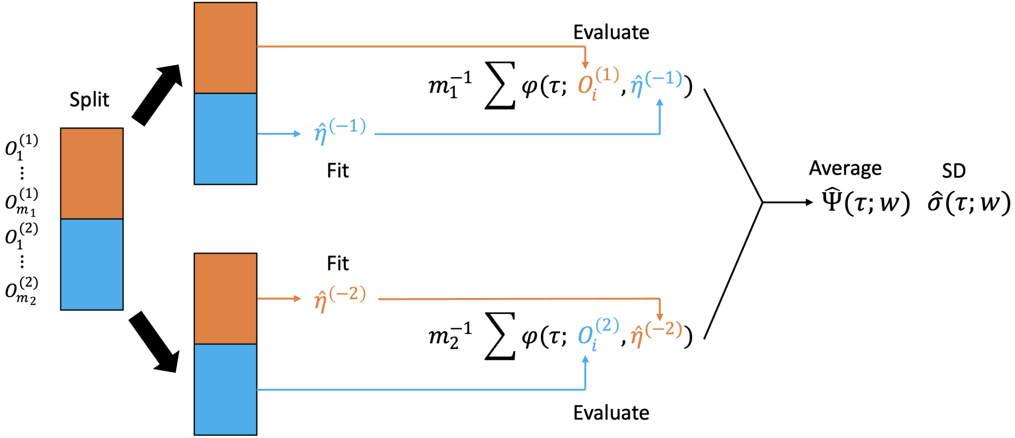
$$\text{IPCW-BC}_{ij}(\tau;\mathbf{O}_i) = \frac{w_j(\mathbf{A}_i,\mathbf{X}_i,N_i)}{\mathbb{P}(\mathbf{A}_i|\mathbf{X}_i,N_i)}\left\{\frac{\Delta_{ij}}{S_{ij}^C(Y_{ij}|\mathbf{A}_i,\mathbf{X}_i,N_i)}\mathbb{1}(Y_{ij}\leq\tau) - F_{ij}^T(\tau|\mathbf{A}_i,\mathbf{X}_i,N_i)\right\},$$

$$\mathrm{AUG}_{ij}(\tau;\mathbf{O}_i) = \frac{w_j(\mathbf{A}_i,\mathbf{X}_i,N_i)}{\mathbb{P}(\mathbf{A}_i|\mathbf{X}_i,N_i)}\int_0^\tau \frac{S_{ij}^T(r|\mathbf{A}_i,\mathbf{X}_i,N_i) - S_{ij}^T(\tau|\mathbf{A}_i,\mathbf{X}_i,N_i)}{S_{ij}^C(r|\mathbf{A}_i,\mathbf{X}_i,N_i)S_{ij}^T(r|\mathbf{A}_i,\mathbf{X}_i,N_i)}dM_{ij}^C(r)$$

Nuisance functions

- Nuisance functions $\boldsymbol{\eta} = (\mathbf{F}^T, \mathbf{S}^C, H, \mathbf{w}, \phi)$:
  1. Event time distribution function $\mathbf{F}^T(r|\mathbf{a}_i, \mathbf{x}_i, n_i) = \left[F_{ij}^T(r|\mathbf{A}_i = \mathbf{a}_i, \mathbf{X}_i = \mathbf{x}_i, N_i = n_i)\right]_{j=1}^{n_i}$
  2. Censoring time survival function $\mathbf{S}^C(r|\mathbf{a}_i, \mathbf{x}_i, n_i) = \left[S_{ij}^C(r|\mathbf{A}_i = \mathbf{a}_i, \mathbf{X}_i = \mathbf{x}_i, N_i = n_i)\right]_{j=1}^{n_i}$
  3. Cluster treatment probability $H(\mathbf{a}_i, \mathbf{x}_i, n_i) = \mathbb{P}\big(\mathbf{A}_i = \mathbf{a}_i | \mathbf{X}_i = \mathbf{x}_i, N_i = n_i\big)$
  4. Weight function $\mathbf{w}(\mathbf{a}_i, \mathbf{x}_i, n_i)$
  5. EIF of the weight function $\phi(\mathbf{a}_i', \mathbf{x}_i, n_i; \mathbf{a}_i)$
- $\mathbf{F}^T \& \mathbf{S}^C$: Random survival forest (Ishwaran et al. 2023)
- H: Random effect Bayesian additive regression trees (Chipman et al. 2010, Dorie 2022)

Sample Splitting Estimator



Theoretical results

Theorem

Under the mild conditions s.t nuisance function estimators have convergence rate of $m^{-1/4}$, *then* $\sqrt{m}\{\widehat{\Psi}(\tau;\mathbf{w}) - \Psi(\tau;\mathbf{w})\}/\widehat{\sigma}(\tau;\mathbf{w}) \xrightarrow{d} N(0,1)$, *where* $\widehat{\sigma}(\tau;\mathbf{w}) \xrightarrow{p} \sigma(\tau;\mathbf{w})$. *Also, if there is no censoring,* $\sigma(\tau;\mathbf{w})^2 = \mathrm{Var}\{\varphi^{F,*}(\tau;\mathbf{O},\boldsymbol{\eta})\}$ *is the nonparametric efficiency bound of* $\Psi(\tau;\mathbf{w})$.

Sketch of proof)

$$\widehat{\Psi}(\tau;\mathbf{w}) - \Psi(\tau;\mathbf{w}) = \frac{1}{K}\sum_{k=1}^{K}\Big[(\mathbb{P}_m^k - \mathbb{P})\varphi(\tau;\mathbf{O};\boldsymbol{\eta}) + (\mathbb{P}_m^k - \mathbb{P})\{\varphi(\tau;\mathbf{O},\widehat{\boldsymbol{\eta}}_{(k)}) - \varphi(\tau;\mathbf{O},\boldsymbol{\eta})\} + \mathbb{P}\{\varphi(\tau;\mathbf{O},\widehat{\boldsymbol{\eta}}_{(k)}) - \varphi(\tau;\mathbf{O},\boldsymbol{\eta})\}\Big]$$

- $(\mathbb{P}_m^k - \mathbb{P})\varphi(\tau;\mathbf{O};\boldsymbol{\eta}) \sim N\big(0,\sigma^2(\tau;\mathbf{w})\big)$: CLT
- $(\mathbb{P}_m^k - \mathbb{P})\{\varphi(\tau;\mathbf{O},\widehat{\boldsymbol{\eta}}_{(k)}) - \varphi(\tau;\mathbf{O},\boldsymbol{\eta})\} = O_{\mathbb{P}}\left(m_k^{-1/2}||\varphi(\tau;\mathbf{O},\widehat{\boldsymbol{\eta}}_{(k)}) - \varphi(\tau;\mathbf{O},\boldsymbol{\eta})||\right)$
- $\mathbb{P}\{\varphi(\tau;\mathbf{O},\widehat{\boldsymbol{\eta}}_{(k)}) - \varphi(\tau;\mathbf{O},\boldsymbol{\eta})\} = O_{\mathbb{P}}\left(r_{\mathbf{w}}^2 + r_{\mathbf{F}^T}(r_H + r_{\phi} + r_{\mathbf{S}^C})\right)$

Theoretical results

Theorem

Under the mild conditions, $\sqrt{m}\{\widehat{\Psi}(\cdot;\mathbf{w}) - \Psi(\cdot;\mathbf{w})\} \rightsquigarrow \mathbb{G}(\cdot)$ *in* $\ell^\infty([0,\tau])$ *as* $m \to \infty$*, where* $\ell^\infty([0,\tau])$ *is a function space with the finite supremum norm over* $[0,\tau]$*, and* $\mathbb{G}(\cdot)$ *is a mean zero Gaussian process with covariance function* $\mathbb{E}\{\mathbb{G}(s)\mathbb{G}(t)\} = \mathrm{Cov}\{\varphi(s;\mathbf{O},\boldsymbol{\eta}), \varphi(t;\mathbf{O},\boldsymbol{\eta})\}$.

Policy	Consistency	Asymptotic Normality	Consistent Variance Estimator	Notes
Type B	$r_{\boldsymbol{\lambda}^T}(r_H + r_{\mathbf{S}^C}) = o(1)$	$r_{\boldsymbol{\lambda}^T}(r_H + r_{\mathbf{S}^C}) = o(m^{-1/2})$	$r_H = r_{\boldsymbol{\lambda}^T} = r_{\mathbf{S}^C} = o(1)$	$\phi = 0$, Doubly Robust
TPB	$r_H = o(1)$, $r_{\boldsymbol{\lambda}^T}(r_H + r_{\mathbf{S}^C}) = o(1)$	$r_H = o(m^{-1/4})$, $r_{\boldsymbol{\lambda}^T}(r_H + r_{\mathbf{S}^C}) = o(m^{-1/2})$	$r_H = r_{\boldsymbol{\lambda}^T} = r_{\mathbf{S}^C} = o(1)$	Consistent *H* estimation required

Simulations

- $D = 1000$ simulations, each consisted of $m = 200$ clusters
- $N_i \overset{iid}{\sim} Unif\{5, 6, \ldots, 20\}, i = 1, \ldots, m$
- $X_{i,c1}, \ldots, X_{i,c5} \overset{iid}{\sim} \text{Unif}(0,1)$: cluster-level covariates
- $X_{ij1}, \ldots, X_{ij5} \overset{iid}{\sim} \text{Unif}(0,1)$, $X_{ij6}, \ldots, X_{ij10} \overset{iid}{\sim} \text{Bernoulli}(0.5)$: individual-level covariates
- $A_{ij} \sim \text{Bernoulli}(\pi_{ij})$: treatment status,
  $\pi_{ij} = \Phi\big(0.1 + 0.2X_{ij1}^2 + 0.2\max\{X_{ij2}, 0.3\}X_{ij6} + 0.3\mathbb{1}(X_{i,c1} > 0.5) + b_i\big)$, $b_i \overset{iid}{\sim} \text{N}(0, 0.5)$
- Event time $T_{ij} \sim \text{Gamma}(0.5A_{ij} + 0.4\overline{\mathbf{A}}_{i(-j)}X_{ij1} + 0.2A_{ij}\overline{\mathbf{A}}_{i(-j)} + 0.2X_{ij2} + 0.4X_{i,c1}, 2)$
- Censoring time $C_{ij} = X_{ij7}C_{ij,\text{Unif}} + (1 - X_{ij7})C_{ij,\text{Poi}}$
- Type B policy with $\alpha \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ at time $\tau = 0.3$

Simulations

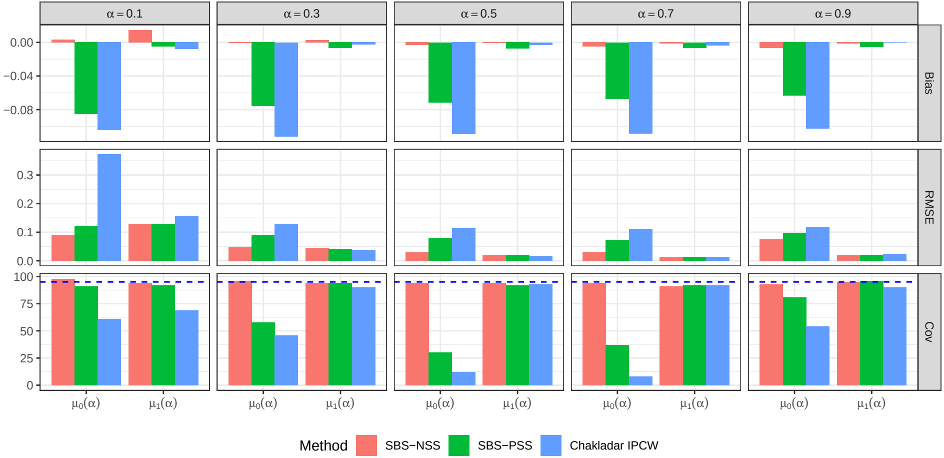
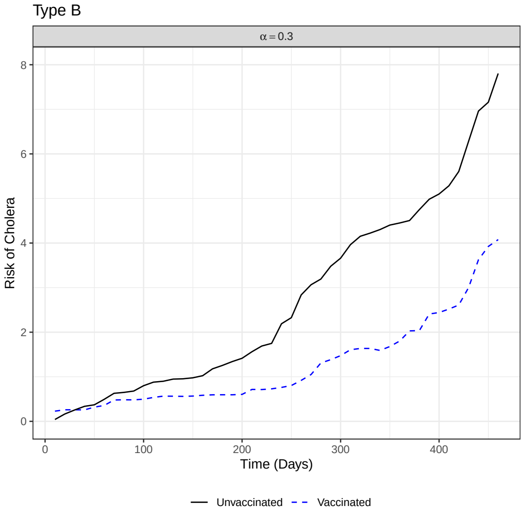


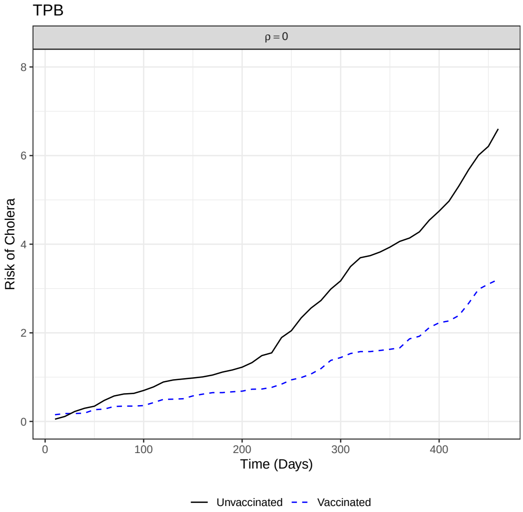
Figure 1: Finite sample performance of the proposed SBS-NSS, SBS-PSS, and Chakladar IPCW estimators for Type B policy; Bias: average bias of estimates, RMSE: root mean squared error, Cov: 95% CI coverage (%)

Application to Cholera Vaccine Study in Bangladesh

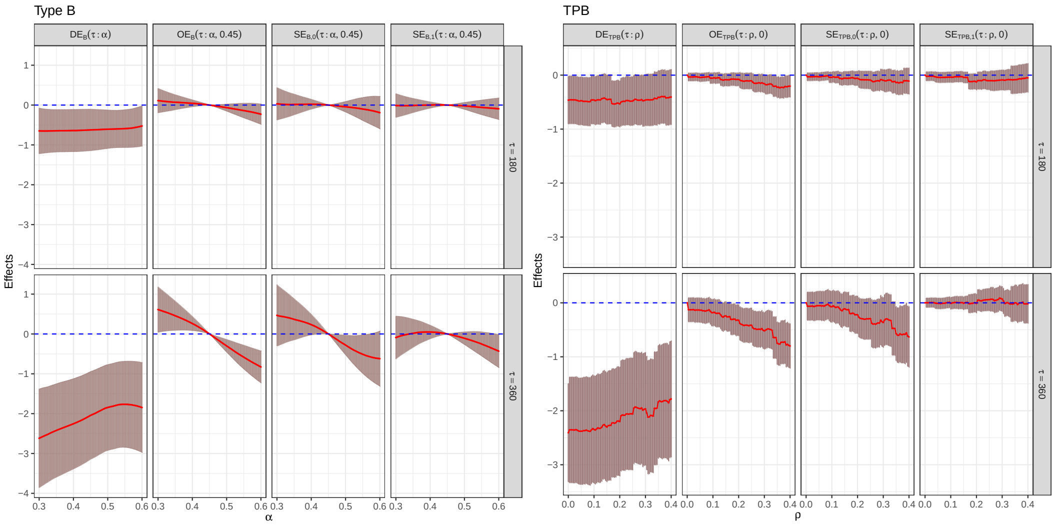
- Prior research suggests possible interference within baris, i.e., clusters of patrilineally related households, but none have utilized data-adaptive methods.
- $m = 5,625$ baris, size of $N_i = 2, \ldots, 239$, total 112,154 individuals.
- Vaccine rate: 45% (48,763 / 112,154)
- Cholera incident rate: 0.4% (458 / 112,154)
- Mean event time: 256 days (IQR: [183, 364])
- Mean censoring time: 412 days (IQR: [397, 431])
- How does cholera incidence change if every individual is vaccinated with a probability of 0.6? [Type B policy]
- How does cholera incidence change if at least 30% of individuals within a bari are vaccinated? [TPB policy]

Application to Cholera Vaccine Study in Bangladesh





Application to Cholera Vaccine Study in Bangladesh



Discussion

- Nonparametric methods are developed which can be used to draw inference about treatment effects in the presence of confounding, clustered interference, and right censoring, and can be applied to any treatment allocation policy, allowing for units' propensity to vary by their covariates and are not based on parametric model
- Proposed nonparametric sample splitting estimators make use of semiparametric efficiency theory and a variety of data-adaptive techniques, and therefore are robust to model mis-specification compared to parametric estimators.
- Application to the cholera vaccine study suggests that vaccination decreases the risk of cholera, and unvaccinated individuals may receive a protective spillover effect from vaccinated individuals. The direct vaccine effect is large when the vaccine coverage is low, while the spillover effect from vaccinated to unvaccinated individuals is large when the vaccine coverage is high.